Risk Measures and Capital Requirements with General Reference Assets

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Risk Day
September 14th 2012, ETH Zürich
Capital adequacy and default risk

- Liability holders of a financial institution are credit sensitive: they, and regulators on their behalf, are concerned that the institution may fail to honor its future obligations.

- This will be the case if the institution’s financial position, i.e. the value of its assets less the value of its liabilities, becomes negative in some future state of the world.

- To address this concern financial institutions hold risk capital, which is meant to absorb unexpected losses, thereby reducing the likelihood that they may become insolvent.

- A key question is how much risk capital a financial institution should be required to hold to be deemed adequately capitalized by the regulator without assuming the availability of a risk-free asset.
Acceptable financial positions

The framework:

- a **single-period** economy with dates $t = 0$ and $t = 1$;
- a **probability space** $(\Omega, \mathcal{F}, \mathbb{P})$: uncertainty at $t = 1$;
- a **Banach lattice** $(\mathcal{X}, \|\cdot\|, \leq)$ of random variables of the form
  $$X : \Omega \to \mathbb{R}$$
  representing the space of financial positions of a given financial institution; for example:
  - the space $L^\infty$ of (essentially) bounded positions;
  - the space $L^p$ of positions having finite $p$-th moment;
  - more general spaces, like Orlicz or Köthe spaces;
- a set of **acceptable positions** $\mathcal{A} \subset \mathcal{X}$ satisfying:
  - $\mathcal{A}$ is a non-empty, proper subset of $\mathcal{X}$ (non-triviality);
  - if $X \in \mathcal{A}$ and $X \leq Y$ then $Y \in \mathcal{A}$ (monotonicity).

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Value-at-Risk and Expected Shortfall

From a regulatory point of view, the most important acceptance sets are the ones based on Value-at-Risk and Expected Shortfall:

(i) the **Value-at-Risk** of a position $X \in \mathcal{X}$ at level $\alpha \in (0, 1)$ is

$$\text{VaR}_\alpha(X) := \inf \{ m \in \mathbb{R} ; \ P(X + m < 0) \leq \alpha \} ,$$

and the corresponding (non-coherent) acceptance set is

$$\mathcal{A}_\alpha := \{ X \in \mathcal{X} ; \text{VaR}_\alpha(X) \leq 0 \} = \{ X \in \mathcal{X} ; \ P(X < 0) \leq \alpha \} ;$$

(ii) the **Expected-Shortfall** of a position $X \in \mathcal{X}$ at level $\alpha \in (0, 1)$ is

$$\text{ES}_\alpha(X) := \frac{1}{\alpha} \int_0^\alpha \text{VaR}_\beta(X) d\beta ,$$

and the corresponding (coherent) acceptance set is

$$\mathcal{A}^\alpha := \{ X \in \mathcal{X} ; \text{ES}_\alpha(X) \leq 0 \} .$$
From unacceptable to acceptable: the monetary case

**Definition**
Let $\mathcal{A} \subset \mathcal{X}$ be an acceptance set. The **monetary capital requirement** for $X \in \mathcal{X}$ associated to $\mathcal{A}$ is defined as

$$\rho_{\mathcal{A}}(X) := \inf \{ m \in \mathbb{R} ; X + m \in \mathcal{A} \} .$$

**Interpretation**
$\rho_{\mathcal{A}}(X)$ represents the “minimum” amount of capital that needs to be raised and invested in **cash** at $t = 0$ in order to reach acceptability.

**Examples**
- $\text{VaR}_\alpha(X) = \inf \{ m \in \mathbb{R} ; \mathbb{P}(X + m < 0) \leq \alpha \}$;
- $\text{ES}_\alpha(X) = \inf \{ m \in \mathbb{R} ; \int_0^\alpha \text{VaR}_\beta(X + m) d\beta \leq 0 \}$.
From unacceptable to acceptable: general reference asset

Definition
Let $A \subset X$ be an acceptance set, and $S$ an asset with

- $t = 0$ price $S_0 = 1$;
- $t = 1$ payoff $S_1 \in X$ with $S_1 \geq 0$.

The **capital requirement** for $X \in X$ associated to $A$ and $S$ is defined as

$$\rho_A, s(X) := \inf \left\{ m \in \mathbb{R} ; X + mS_1 \in A \right\} .$$

Interpretation
$\rho_A, s(X)$ represents the “minimum” amount of capital that needs to be raised and invested at $t = 0$ in the reference asset $S$ in order to reach acceptability.

Examples

- $\rho_{A\alpha}, s(X) = \inf \left\{ m \in \mathbb{R} ; \mathbb{P}(X + mS_1 < 0) \leq \alpha \right\} ;$
- $\rho_{A\alpha}, s(X) = \inf \left\{ m \in \mathbb{R} ; \int_0^\alpha \text{VaR}_\beta(X + mS_1) d\beta \leq 0 \right\} . $
General properties of capital requirements

Proposition

Let $\mathcal{A} \subseteq \mathcal{X}$ be an acceptance set, and $S$ the reference asset. Then

(i) $\rho_{\mathcal{A},S}$ is $S$-additive:

$$\rho_{\mathcal{A},S}(X + mS_1) = \rho_{\mathcal{A},S}(X) - m \quad \text{for all } X \in \mathcal{X}, m \in \mathbb{R};$$

(ii) $\rho_{\mathcal{A},S}$ is monotonic (decreasing):

$$\rho_{\mathcal{A},S}(X) \geq \rho_{\mathcal{A},S}(Y) \quad \text{for all } X, Y \in \mathcal{X}, X \leq Y.$$

General capital requirements $\rho_{\mathcal{A},S}$ do not satisfy:

- the axiom of cash-additivity (Artzner et al. (1999))

  $$\rho_{\mathcal{A},S}(X - m) = \rho_{\mathcal{A},S}(X) + m \quad \text{for all } X \in \mathcal{X}, m \in \mathbb{R};$$

- the axiom of cash-subadditivity (El Karoui-Ravanelli (2009) and Cerreia-Vioglio et al. (2011))

  $$\rho_{\mathcal{A},S}(X - m) \geq \rho_{\mathcal{A},S}(X) + m \quad \text{for all } X \in \mathcal{X}, m > 0.$$
Why is $\rho_{\mathcal{A},S}$ worth studying?

- Considering reference assets other than cash might allow to reach acceptability at a **lower cost**.

- If the payoff $S_1$ is bounded away from zero, i.e. $S_1 \geq \varepsilon$ for some $\varepsilon > 0$, and we consider the **discounted** acceptance set

\[
\mathcal{A}_S := \{X / S_1 \in \mathcal{X} ; X \in \mathcal{A}\},
\]

then $\rho_{\mathcal{A},S}(X) = \rho_{\mathcal{A}_S}(X / S_1)$.

- If $S_1$ is not bounded away from zero, then typically $\rho_{\mathcal{A},S}$ can no longer be reduced to a monetary capital requirement by choosing $S$ as the **numeraire**:
  - the discounted position $X / S_1$ might lie **outside** $\mathcal{X}$;
  - the payoff $S_1$ might be **zero** in some future state of the economy.

- We allow for general, possibly **defaultable**, reference assets: stocks (e.g. modeled by lognormal distributions), options, defaultable bonds.
Effectiveness and robustness

Since the standard theory of monetary risk measures cannot be applied, we investigate the following questions.

- When is required capital a well-defined number for any financial position (effectiveness)?
  - If $\rho_{\mathcal{A},S}(X) = \infty$, then $X$ cannot be made acceptable by investing any amount of capital in the reference asset, suggesting $S$ is not a good vehicle to reach acceptability.
  - If $\rho_{\mathcal{A},S}(X) = -\infty$, we can extract arbitrary amounts of capital going short on $S$ without compromising the acceptability of $X$, a situation that is not plausible.

- When is required capital a continuous function of financial positions (robustness)?
  - If $\rho_{\mathcal{A},S}$ is not continuous at $X$, then a slight change in our portfolio might lead to a dramatical change in the corresponding capital requirement.
Lack of effectiveness and robustness: VaR

Let $\mathcal{X} = L^p$ with $0 \leq p \leq \infty$ on a nonatomic probability space $(\Omega, \mathcal{F}, \mathbb{P})$. Recall the acceptance set based on Value-at-Risk at level $0 < \alpha < 1$

$$\mathcal{A}_\alpha := \{ X \in L^p ; \mathbb{P}(X < 0) \leq \alpha \} .$$

Proposition

Let $0 < \alpha < 1$ and $S$ be the reference asset.

(a) Assume $p = \infty$. Then the following statements hold:

(i) $\rho_{\mathcal{A}_\alpha, S}$ is effective $\iff$ $\text{VaR}_\alpha(S_1) < 0 < \text{VaR}_\alpha(-S_1)$;

(ii) $\rho_{\mathcal{A}_\alpha, S}$ is robust $\iff$ $S_1 \geq \varepsilon$ for some $\varepsilon > 0$.

(b) Assume $p < \infty$. Then the following statements hold:

(i) $\rho_{\mathcal{A}_\alpha, S}$ is effective $\iff$ $\mathbb{P}(S_1 = 0) < \min\{\alpha, 1 - \alpha\}$;

(ii) $\rho_{\mathcal{A}_\alpha, S}$ is never robust on the whole $L^p$;

(iii) $\text{VaR}_\alpha$ is continuous at $X$ $\iff$ the lower and upper $\alpha$-quantile of $X$ coincide.
Lack of effectiveness and robustness: ES

Let \( \mathcal{X} = L^p \) with \( 1 \leq p \leq \infty \) on a nonatomic probability space \( (\Omega, \mathcal{F}, \mathbb{P}) \). Recall the acceptance set based on Expected Shortfall at level \( 0 < \alpha < 1 \)

\[
\mathcal{A}^\alpha := \{ X \in L^p ; \int_0^\alpha \text{VaR}_\beta(X) d\beta \leq 0 \} .
\]

Proposition

Let \( 0 < \alpha < 1 \) and \( S \) be the reference asset. The following statements are equivalent:

(i) \( \rho_{\mathcal{A}^\alpha, S} \) is effective;

(ii) \( \rho_{\mathcal{A}^\alpha, S} \) is robust (Lipschitz continuous);

(iii) there exists \( \lambda > 0 \) such that \( \mathbb{P}(S_1 < \lambda) < \alpha \);

(iv) \( \text{ES}_\alpha(S_1) < 0 \).
Proposition

Let $\mathcal{X}_+ := \{X \in \mathcal{X} ; X \geq 0\}$ be the positive cone of $\mathcal{X}$. Let $\mathcal{A} \subset \mathcal{X}$ be an arbitrary acceptance set and $S$ the reference asset. If the payoff $S_1$ is an interior point of $\mathcal{X}_+$, then:

(i) $\rho_{\mathcal{A},S}$ is finitely valued and continuous on $\mathcal{X}$;

(ii) if $\mathcal{A}$ is convex, then $\rho_{\mathcal{A},S}$ is locally Lipschitz continuous;

(iii) if $\mathcal{A}$ is a convex cone (coherent), then $\rho_{\mathcal{A},S}$ is globally Lipschitz continuous.

Examples

- The positive cone of $L^\infty$ has non-empty interior and
  \[ \text{int}(L^\infty) = \{X \in L^\infty ; \exists \varepsilon > 0 : X \geq \varepsilon\} . \]

- The positive cone of $L^p$ for $p < \infty$ or general Orlicz spaces has empty interior.
Strictly positive payoffs

Definition

A position \( X \in \mathcal{X}_+ \) is **strictly positive** if \( \psi(X) > 0 \) for every linear continuous functional \( \psi : \mathcal{X} \rightarrow \mathbb{R} \).

Examples

- If \( \mathcal{X} := L^\infty \) then strictly positive positions and interior points of \( L^\infty_+ \) coincide.
- If \( \mathcal{X} := L^p \) with \( 1 \leq p < \infty \), then strictly positive positions are \( X \in L^p \) such that \( X > 0 \) almost surely.

Theorem

Let \( \mathcal{A} \subset \mathcal{X} \) be an acceptance set with nonempty interior and \( S \) the reference asset. Assume that \( \rho_{\mathcal{A}, S} \) does not attain the value \(-\infty\).

If the payoff \( S_1 \) is **strictly positive**, then

1. \( \rho_{\mathcal{A}, S} \) is finitely valued;
2. if \( \mathcal{A} \) is convex, then \( \rho_{\mathcal{A}, S} \) is also (Lipschitz) continuous.
Conic and coherent acceptance sets

Proposition

Let \( A \subset \mathcal{X} \) be a conic acceptance set with nonempty interior and \( S \) the reference asset. The following statements hold:

(i) \( \rho_A, S < \infty \iff S_1 \in \text{int}(A) \);

(ii) \( \rho_A, S > -\infty \iff -S_1 \in \text{int}(A^c) \).

If we additionally require coherence, we can show that effectiveness and robustness are indeed equivalent.

Theorem

Let \( A \subset \mathcal{X} \) be a coherent acceptance set with nonempty interior and \( S \) the reference asset. The following statements are equivalent:

(i) \( \rho_A, S \) is finitely valued;

(ii) \( \rho_A, S \) is continuous;

(iii) \( S_1 \in \text{int}(A) \).
Intermediate summary

The capital requirement of a position $X \in \mathcal{X}$ based on $\mathcal{A}$ and $S$ (with price $S_0 = 1$ and payoff $S_1 \in \mathcal{X}_+$) has been defined as

$$\rho_{\mathcal{A},S}(X) := \inf \{ m \in \mathbb{R} ; X + mS_1 \in \mathcal{A} \} .$$

Main issues:

- the asset $S$ is not assumed to be risk-free: $S$ might describe a stock, an option, a zero-coupon bond under stochastic interest rate, a general defaultable security;

- beyond the theory of monetary risk measures and cash-subadditive risk measures;

- finiteness and continuity as a result of the interplay between $\mathcal{A}$ and $S$;

- general results applicable to explicit capital requirements (e.g. based on VaR or ES);

- all results can be extended to the context of a general ordered topological vector space.
Multiple reference assets: underlying model

The framework:

- a **single-period** economy with dates $t = 0$ and $t = 1$;
- a **probability space** $(\Omega, \mathcal{F}, \mathbb{P})$;
- a **Banach lattice** $(\mathcal{X}, \| \cdot \|, \leq)$ of random variables of the form $X : \Omega \rightarrow \mathbb{R}$ representing the space of financial positions;
- a set of **acceptable positions** $\mathcal{A} \subset \mathcal{X}$ satisfying:
  - $\mathcal{A}$ is a non-empty, proper subset of $\mathcal{X}$ (non-triviality);
  - if $X \in \mathcal{A}$ and $X \leq Y$ then $Y \in \mathcal{A}$ (monotonicity).
Multi-asset capital requirements

We allow to modify the acceptability of a financial position by investing in portfolios of \( N \) reference assets \( S^{(1)}, \ldots, S^{(N)} \) having:

- initial price \( S_0^{(i)} = 1 \);
- terminal payoff \( S_1^{(i)} \in \mathcal{X} \) with \( S_1^{(i)} \geq 0 \);
- \( \mathcal{M}(\mathcal{I}) := \left\{ \sum_{i=1}^{N} \lambda_i S_1^{(i)} ; \lambda_1, \ldots, \lambda_N \in \mathbb{R} \right\} \) is the marketed space or hedging space generated by \( \mathcal{I} := \{ S^{(1)}, \ldots, S^{(N)} \} \);
- \( \pi \left( \sum_{i=1}^{N} \lambda_i S_1^{(i)} \right) := \sum_{i=1}^{N} \lambda_i \) is the pricing functional on \( \mathcal{M}(\mathcal{I}) \).

Definition

Let \( \mathcal{A} \subset \mathcal{X} \) be an acceptance set and \( \mathcal{I} := \{ S^{(1)}, \ldots, S^{(N)} \} \).

The capital requirement of \( X \in \mathcal{X} \) associated to \( \mathcal{A} \) and \( \mathcal{I} \) is defined as

\[
\rho_{\mathcal{A},\mathcal{I}}(X) := \inf \left\{ \pi(Z) \in \mathbb{R} ; Z \in \mathcal{M}(\mathcal{I}) : X + Z \in \mathcal{A} \right\}.
\]
Why multiple reference assets?

Consider an acceptance set $\mathcal{A} \subset X$ and a set of reference assets $\mathcal{S} = \{S^{(1)}, \cdots, S^{(N)}\}$.

The multi-asset capital requirement of $X \in X$ has been defined as

$$\rho_{\mathcal{A},\mathcal{S}}(X) = \inf \left\{ \sum_{i=1}^{N} \lambda_i ; X + \sum_{i=1}^{N} \lambda_i S^{(i)} \in \mathcal{A} \right\}.$$

Motivation

- **Capital efficiency**: for every $S \in \mathcal{S}$ and $X \in X$

  $$\rho_{\mathcal{A},\mathcal{S}}(X) \leq \rho_{\mathcal{A},s}(X).$$

- **Multi-currency setting**: the assets $S^{(1)}, \cdots, S^{(N)}$ may be regarded as risk-free assets corresponding to different currencies.
Proposition

Let \( A \subset X \) be an acceptance set, and \( I \) the class of reference assets. Then

(i) \( \rho_{A,I} \) is \( I \)-additive:

\[
\rho_{A,I}(X + Z) = \rho_{A,I}(X) - \pi(Z)
\]

for all \( X \in X \), \( Z \in \mathcal{M}(I) \);

(ii) \( \rho_{A,I} \) is monotonic (decreasing):

\[
\rho_{A,I}(X) \geq \rho_{A,I}(Y)
\]

for all \( X, Y \in X \), \( X \leq Y \);

(iii) if \( I = \{S\} \) then \( \rho_{A,I} = \rho_{A,S} \).
From several assets to a single asset

**Lemma (Reduction Lemma)**

Let \( A \subset X \) be an acceptance set and let \( M(S) \) be the hedging space. Define

\[
M_0(S) := \{ Z \in M(S) ; \pi(Z) = 0 \}.
\]

Then for any asset \( S \in \mathcal{I} \) we get

\[
\rho_A, S = \rho_A + M_0(S), S.
\]

**Remark**

- \( A + M_0(S) \neq X \implies A + M_0(S) \) is an acceptance set;
- \( A + M_0(S) = X \implies \rho_A, S \equiv -\infty \), i.e.

for every \( X \in X \) there exists \( Z \in M_0(S) \) such that \( X + Z \in A \).
Absence of acceptability arbitrage

Definition
Let $\mathcal{A} \subset \mathcal{X}$ be an acceptance set and $\mathcal{M}(\mathcal{I})$ the hedging space. The condition of **absence of acceptability arbitrage** holds if

$$\mathcal{A} + \mathcal{M}_0(\mathcal{I}) \neq \mathcal{X} \quad (\text{NAA}(\mathcal{A}, \mathcal{I})) .$$

Theorem
Let $\mathcal{A} \subset \mathcal{X}$ be an acceptance set with nonempty interior and $\mathcal{M}(\mathcal{I})$ the hedging space. Set $\mathcal{M}_m(\mathcal{I}) := \{ Z \in \mathcal{M}(\mathcal{I}); \pi(Z) = m \}$.

The following statements hold:

(i) $\mathcal{A} \cap \mathcal{M}_m(\mathcal{I}) = \emptyset$ for some $m \in \mathbb{R} \implies \mathcal{A} \cap \mathcal{M}_k(\mathcal{I}) = \emptyset$ for $k < m \implies \text{NAA}(\mathcal{A}, \mathcal{I})$;

(ii) $\mathcal{A}$ is convex and $\text{int}(\mathcal{A}) \cap \mathcal{M}_0(\mathcal{I}) = \emptyset \implies \text{NAA}(\mathcal{A}, \mathcal{I})$;

(iii) $\mathcal{A}$ is a cone and $\text{NAA}(\mathcal{A}, \mathcal{I}) \implies \text{int}(\mathcal{A}) \cap \mathcal{M}_0(\mathcal{I}) = \emptyset$;

(iv) if $\mathcal{A}$ is coherent then $\text{NAA}(\mathcal{A}, \mathcal{I}) \iff \text{int}(\mathcal{A}) \cap \mathcal{M}_0(\mathcal{I}) = \emptyset$. 
Robust representations of capital requirements

For simplicity let $\mathcal{X} := L^\infty$ be the space of (essentially) bounded random variables over $(\Omega, \mathcal{F}, \mathbb{P})$.

**Theorem**

Let $\mathcal{A} \subset L^\infty$ be a closed, convex acceptance set and $\mathcal{M}(\mathcal{I})$ the hedging space satisfying $\text{NAA} (\mathcal{A}, \mathcal{I})$. Define

$$\mathcal{D}(\mathcal{I}) := \{ Q : \mathcal{F} \rightarrow [0, 1] \text{ finitely additive}; Q << \mathbb{P},$$

$$\mathbb{E}_Q[S^{(i)}_1] = \pi(S^{(i)}_1), \; i = 1, \ldots, N\}.$$  

Then for every $X \in \mathcal{X}$

$$\rho_{\mathcal{A}, \mathcal{I}}(X) = \sup_{Q \in \mathcal{D}(\mathcal{I})} \{ \sigma_{\mathcal{A}}(Q) - \mathbb{E}_Q[X] \},$$

where $\sigma_{\mathcal{A}}(Q) := \inf_{Y \in \mathcal{A}} \mathbb{E}_Q[Y]$ is the support function of $\mathcal{A}$. 
Optimal capital allocation

Definition
Let $\rho_1, \cdots, \rho_N$ be capital requirements, and take $X \in \mathcal{X}$. The infimal convolution of $\rho_1, \cdots, \rho_N$ at $X$ is defined as

$$
\text{conv}(\rho_1, \cdots, \rho_N)(X) := \inf \left\{ \sum_{i=1}^{N} \rho_i(X_i) ; \ X_i \in \mathcal{X} : \sum_{i=1}^{N} X_i = X \right\}.
$$

Proposition
Let $\mathcal{A}_1, \cdots, \mathcal{A}_N \subset \mathcal{X}$ be acceptance sets and choose a set of reference assets $\mathcal{S} := \{S^{(1)}, \cdots, S^{(N)}\}$. Then for all $X \in \mathcal{X}$

$$
\text{conv}(\rho_{\mathcal{A}_1, S^{(1)}}, \cdots, \rho_{\mathcal{A}_N, S^{(N)}})(X) = \rho_{\mathcal{A}_1 + \cdots + \mathcal{A}_N, \mathcal{S}}(X).
$$

If $\mathcal{A} \subset \mathcal{X}$ is a coherent acceptance set, then

$$
\text{conv}(\rho_{\mathcal{A}, S^{(1)}}, \cdots, \rho_{\mathcal{A}, S^{(N)}})(X) = \rho_{\mathcal{A}, \mathcal{S}}(X).
$$
Final summary

- The **capital requirement** for a position $X \in \mathcal{X}$ based on the acceptance set $\mathcal{A} \subset \mathcal{X}$ and the reference assets

$$\mathcal{S} = \{S^{(1)}, \ldots, S^{(N)}\}$$

($S^{(i)}$ having price 1 and payoff $S_1^{(i)}$) is

$$\rho_{\mathcal{A}, \mathcal{S}}(X) = \inf \left\{ \sum_{i=1}^{N} \lambda_i ; X + \sum_{i=1}^{N} \lambda_i S_1^{(i)} \in \mathcal{A} \right\}.$$

- The **reduction lemma**: if $\mathcal{M}_0(\mathcal{S})$ is the set of hedging positions with zero cost, then for every $S \in \mathcal{S}$

$$\rho_{\mathcal{A}, \mathcal{S}} = \rho_{\mathcal{A} + \mathcal{M}_0(\mathcal{S}), \mathcal{S}}.$$

- **Absence of acceptability arbitrage**:

$$\mathcal{A} + \mathcal{M}_0(\mathcal{S}) \neq \mathcal{X}.$$

- **Representation** via scenario measures and infimal convolutions.
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Thank you for your attention!